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```
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\*.qts

\*.qts

Operator \* for qts Objects

#### **Description**

This function implements the pointwise quaternion Hamilton multiplication between two quaternion time series.

## Usage

```
## S3 method for class 'qts'
x * rhs
```

## **Arguments**

x An object of class qts.

rhs Either an object of class qts or a numeric value.

#### Value

An object of class qts storing the multiplication of the two inputs.

## **Examples**

```
vespa64$igp[[1]] * vespa64$igp[[2]]
```

+.qts

*Operator + for* qts *Objects* 

## **Description**

This function implements the pointwise addition between two quaternion time series.

## Usage

```
## S3 method for class 'qts'
x + rhs
```

#### **Arguments**

x An object of class qts.

rhs Either an object of class qts or a numeric value.

## Value

An object of class qts storing the addition of the two inputs.

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#### **Examples**

```
vespa64$igp[[1]] + vespa64$igp[[2]]
```

-.qts

Operator - for qts Objects

## **Description**

This function implements the pointwise subtraction between two quaternion time series.

## Usage

```
## S3 method for class 'qts'
x - rhs
```

## **Arguments**

x An object of class qts.

rhs Either an object of class qts or a numeric value.

#### Value

An object of class qts storing the subtraction of the two inputs.

## **Examples**

```
vespa64$igp[[1]] - vespa64$igp[[2]]
```

append

QTS Sample Concatenation

## Description

QTS Sample Concatenation

## Usage

```
append(x, ...)
## Default S3 method:
append(x, values, after = length(x), ...)
## S3 method for class 'qts_sample'
append(x, y, ...)
```

autoplot.prcomp\_qts 5

## **Arguments**

x	Either a numeric vector or an object of class qts_sample.	
	Extra arguments to be passed on to next methods.	
values	to be included in the modified vector.	
after	a subscript, after which the values are to be appended.	
У	Either a numeric vector or an object of class qts_sample or an object of class	

## Value

If x is a numeric vector, the output is a numeric vector containing the values in x with the elements of values appended after the specified element of x. If x is of class  $qts\_sample$ , the output is another object of class  $qts\_sample$  containing the elements in x and the ones in y appended after the last element of x.

## Examples

#### **Description**

This function creates a visualization of the results of the PCA applied on a sample of QTS and returns the corresponding ggplot2::ggplot object which enable further customization of the plot.

## Usage

```
## S3 method for class 'prcomp_qts'
autoplot(object, what = "PC1", ...)
```

qts.

## Arguments

object	An object of class $prcomp_qts$ as $produced$ by the $prcomp.qts_sample()$ method.
what	A string specifying what kind of visualization the user wants to perform. Choices are words starting with PC and ending with a PC number (in which case the mean QTS is displayed along with its perturbations due to the required PC) or scores (in which case individuals are projected on the required plane). Defaults to PC1.
	If what = "PC?", the user can specify whether to plot the QTS in the tangent space or in the original space by providing a boolean argument original_space which defaults to TRUE. If what = "scores", the user can specify the plane onto which the individuals will be projected by providing a length-2 integer vector argument plane which defaults to 1:2.

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#### Value

```
A ggplot2::ggplot object.
```

## **Examples**

```
df <- as_qts_sample(vespa64$igp[1:16])
res_pca <- prcomp(df)

# Plot the data points in a PC plane
# And color points according to a categorical variable
p <- ggplot2::autoplot(res_pca, what = "scores")
p + ggplot2::geom_point(ggplot2::aes(color = vespa64$V[1:16]))</pre>
```

autoplot.qts

Plot for qts objects

## **Description**

This function creates a visualization of a QTS and returns the corresponding ggplot2::ggplot object which enable further customization of the plot.

#### Usage

```
## S3 method for class 'qts'
autoplot(object, highlighted_points = NULL, ...)
```

## **Arguments**

object An object of class qts. highlighted\_points

An integer vector specifying point indices to be highlighted. Defaults to NULL, in which case no point will be highlighted with respect to the others.

Further arguments to be passed on to next methods.

#### Value

```
A ggplot2::ggplot object.
```

```
ggplot2::autoplot(vespa64$igp[[1]])
```

autoplot.qtsclust 7

autoplot.qtsclust

Plot for qtsclust objects

#### **Description**

This function creates a visualization of the clustering results obtained on a sample of QTS and returns the corresponding ggplot2::ggplot object which enable further customization of the plot.

## Usage

```
## S3 method for class 'qtsclust'
autoplot(object, ...)
```

## Arguments

object An object of class qtsclust as produced by kmeans.qts\_sample() or hclust.qts\_sample().
... Further arguments to be passed to other methods.

#### Value

A ggplot2::ggplot object.

## **Examples**

```
out <- kmeans(vespa64$igp[1:10], n_clusters = 2)
ggplot2::autoplot(out)</pre>
```

```
autoplot.qts_sample Plot for qts_sample objects
```

## Description

This function creates a visualization of a sample of QTS and returns the corresponding ggplot2::ggplot object which enable further customization of the plot.

## Usage

```
## $3 method for class 'qts_sample'
autoplot(
  object,
  memberships = NULL,
  highlighted = NULL,
  with_animation = FALSE,
  ...
)
```

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#### **Arguments**

object An object of class qts\_sample.

memberships A vector coercible as factor specifying a group membership for each QTS in the

sample. Defaults to NULL, in which case no grouping structure is displayed.

highlighted A boolean vector specifying whether each QTS in the sample should be hight-

lighted. Defaults to NULL, in which case no QTS is hightlighted w.r.t. the others.

with\_animation A boolean value specifying whether to create a an animated plot or a static gg-

plot2::ggplot object. Defaults to FALSE which will create a static plot.

... Further arguments to be passed to methods.

#### Value

A ggplot2::ggplot object.

## **Examples**

```
ggplot2::autoplot(vespa64$igp)
```

centring

QTS Centering and Standardization

## **Description**

This function operates a centering of the QTS around the geometric mean of its quaternions. This is effectively achieved by left-multiplying each quaternion by the inverse of their geometric mean.

#### Usage

```
centring(x, standardize = FALSE, keep_summary_stats = FALSE)
```

## **Arguments**

x An object of class qts.

standardize A boolean specifying whether to standardize the QTS in addition to centering it.

Defaults to FALSE.

keep\_summary\_stats

A boolean specifying whether the mean and standard deviation used for standardizing the data should be stored in the output object. Defaults to FALSE in which case only the centered qts is returned.

dbscan 9

#### Value

If keep\_summary\_stats = FALSE, an object of class qts in which quaternions have been centered (and possibly standardized) around their geometric mean. If keep\_summary\_stats = TRUE, a list with three components:

- qts: an object of class qts in which quaternions have been centered (and possibly standardized) around their geometric mean;
- mean: a numeric vector with the quaternion Fréchet mean;
- sd: a numeric value with the quaternion Fréchet standard deviation.

## **Examples**

```
centring(vespa64$igp[[1]])
```

dbscan

QTS Nearest-Neighbor Clustering

#### Description

This function massages the input quaternion time series to apply DBSCAN clustering on them, with the possibility of separating amplitude and phase variability and of choosing the source of variability through which clusters should be searched.

#### Usage

```
dbscan(x, ...)
## Default S3 method:
dbscan(x, eps, minPts = 5, weights = NULL, borderPoints = TRUE, ...)
## S3 method for class 'qts_sample'
dbscan(
    x,
    warping_class = c("affine", "dilation", "none", "shift", "srsf"),
    centroid_type = "mean",
    metric = c("12", "pearson"),
    cluster_on_phase = FALSE,
    use_fence = FALSE,
    ...
)
```

#### **Arguments**

Х

Either a numeric matrix of data, or an object that can be coerced to such a matrix (such as a numeric vector or a data frame with all numeric columns) or an object of class qts\_sample.

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... additional arguments are passed on to the fixed-radius nearest neighbor search

algorithm. See frNN() for details on how to control the search strategy.

eps size (radius) of the epsilon neighborhood. Can be omitted if x is a frNN object.

minPts number of minimum points required in the eps neighborhood for core points

(including the point itself).

weights numeric; weights for the data points. Only needed to perform weighted cluster-

ing.

borderPoints logical; should border points be assigned to clusters. The default is TRUE for

regular DBSCAN. If FALSE then border points are considered noise (see DB-

SCAN\* in Campello et al, 2013).

warping\_class A string specifying the warping class Choices are "affine", "dilation", "none",

"shift" or "srsf". Defaults to "affine". The SRSF class is the only class

which is boundary-preserving.

centroid\_type A string specifying the type of centroid to compute. Choices are "mean", "median"

"medoid", "lowess" or "poly". Defaults to "mean". If LOWESS appproximation is chosen, the user can append an integer between 0 and 100 as in "lowess20". This number will be used as the smoother span. This gives the proportion of points in the plot which influence the smooth at each value. Larger values give more smoothness. The default value is 10%. If polynomial approximation is chosen, the user can append an positive integer as in "poly3". This number will be used as the degree of the polynomial model. The default value

is 4L.

metric A character string specifying the distance measure to be used. This must be

one of "euclidean", "maximum", "manhattan", "canberra", "binary" or "minkowski" if x is not a QTS sample. Otherwise, it must be one of "l2",

"pearson" or "dtw".

cluster\_on\_phase

A boolean specifying whether clustering should be based on phase variation or

amplitude variation. Defaults to FALSE which implies amplitude variation.

use\_fence A boolean specifying whether the fence algorithm should be used to robus-

tify the algorithm against outliers. Defaults to FALSE. This is used only when warping\_class != "srsf".

#### Value

An object of class stats::kmeans or stats::hclust or dbscan\_fast if the input x is NOT of class qts\_sample. Otherwise, an object of class qtsclust which is effectively a list with four components:

- qts\_aligned: An object of class qts\_sample storing the sample of aligned QTS;
- qts\_centers: A list of objects of class qts representing the centers of the clusters;
- best\_clustering: An object of class fdacluster::caps storing the results of the best k-mean alignment result among all initialization that were tried.
- call\_name: A string storing the name of the function that was used to produce the clustering structure:
- call\_args: A list containing the exact arguments that were passed to the function call\_name that produced this output.

differentiate 11

## **Examples**

```
out <- dbscan(vespa64$igp[1:10])
plot(out)</pre>
```

differentiate

QTS Differentiation

## Description

This function computes the first derivative of quaternion time series with respect to time.

#### Usage

```
differentiate(x)
## S3 method for class 'qts'
differentiate(x)
## S3 method for class 'qts_sample'
differentiate(x)
```

## **Arguments**

Х

An object of class qts or qts\_sample.

#### Value

An object of the same class as the input argument x in which quaternions measure the rotation to be applied to transform attitude at previous time point to attitude at current time point.

## Examples

```
differentiate(vespa64$igp[[1]])
differentiate(vespa64$igp)
```

dist

QTS Distance Matrix Computation

## Description

This function massages an input sample of quaternion time series to turn it into a pairwise distance matrix.

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#### Usage

```
dist(x, metric, ...)
## Default S3 method:
dist(
 х,
 metric = c("euclidean", "maximum", "manhattan", "canberra", "binary", "minkowski"),
 diag = FALSE,
 upper = FALSE,
  p = 2,
)
## S3 method for class 'qts_sample'
dist(
 metric = c("12", "pearson", "dtw"),
 warping_class = c("affine", "dilation", "none", "shift", "srsf"),
  cluster_on_phase = FALSE,
  labels = NULL,
)
```

#### **Arguments**

metric

A numeric matrix, data frame, stats::dist object or object of class qts\_sample specifying the sample on which to compute the pairwise distance matrix.

A character string specifying the distance measure to be used. This must be one of "euclidean", "maximum", "manhattan", "canberra", "binary" or "minkowski" if x is not a QTS sample. Otherwise, it must be one of "12",

"pearson" or "dtw".

... not used.

diag logical value indicating whether the diagonal of the distance matrix should be

printed by print.dist.

upper logical value indicating whether the upper triangle of the distance matrix should

be printed by print.dist.

p The power of the Minkowski distance.

warping\_class A string specifying the warping class Choices are "affine", "dilation", "none",

"shift" or "srsf". Defaults to "affine". The SRSF class is the only class

which is boundary-preserving.

cluster\_on\_phase

A boolean specifying whether clustering should be based on phase variation or amplitude variation. Defaults to FALSE which implies amplitude variation.

labels A character vector specifying curve labels. Defaults to NULL which uses sequen-

tial numbers as labels.

DTW 13

#### Value

An object of class stats::dist.

#### **Examples**

```
D <- dist(vespa64$igp[1:5])</pre>
```

DTW

Dynamic Time Warping for Quaternion Time Series

#### **Description**

This function evaluates the Dynamic Time Warping (DTW) distance between two quaternion time series (QTS).

## Usage

```
DTW(
   qts1,
   qts2,
   resample = TRUE,
   disable_normalization = FALSE,
   distance_only = FALSE,
   step_pattern = dtw::symmetric2)
```

#### **Arguments**

qts1 An object of class qts. qts2 An object of class qts.

resample A boolean specifying whether the QTS should be uniformly resampled on their

domain before computing distances. Defaults to TRUE.

disable\_normalization

A boolean specifying whether quaternion normalization should be disabled. De-

faults to FALSE which ensures that we always deal with unit quaternions.

distance\_only A boolean specifying whether to only compute distance (no backtrack, faster).

Defaults to FALSE.

step\_pattern A dtw::stepPattern specifying the local constraints on the warping path. Defaults

to dtw::symmetric2 which uses symmetric and normalizable warping paths with

no local slope constraints. See dtw::stepPattern for more information.

#### Details

If no evaluation grid is provided, the function assumes that the two input QTS are evaluated on the same grid.

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## Value

An object of class dtw::dtw storing the dynamic time warping results.

#### **Examples**

```
DTW(vespa64$igp[[1]], vespa64$igp[[2]])
```

exp

QTS Exponential

## Description

This function computes the exponential of quaternion time series as the time series of the quaternion exponentials.

## Usage

```
## $3 method for class 'qts'
exp(x, ...)
## $3 method for class 'qts_sample'
exp(x, ...)
```

#### **Arguments**

- x An object of class qts or qts\_sample.
- Extra arguments to be passed on to next methods.

#### Value

An object of the same class as the input argument x in which quaternions have been replaced by their exponential.

```
x <- log(vespa64$igp[[1]])
exp(x)
y <- log(vespa64$igp)
exp(y)</pre>
```

hclust 15

hclust

QTS Hierarchical Agglomerative Clustering

#### **Description**

This function massages the input quaternion time series to apply hierarchical agglomerative clustering on them, with the possibility of separating amplitude and phase variability and of choosing the source of variability through which clusters should be searched.

## Usage

```
hclust(x, metric, linkage_criterion, ...)
## Default S3 method:
hclust(
  х,
 metric = c("euclidean", "maximum", "manhattan", "canberra", "binary", "minkowski"),
  linkage_criterion = c("complete", "average", "single", "ward.D2"),
)
## S3 method for class 'qts_sample'
hclust(
 metric = c("12", "pearson"),
  linkage_criterion = c("complete", "average", "single", "ward.D2"),
  n_{clusters} = 1L,
  warping_class = c("affine", "dilation", "none", "shift", "srsf"),
  centroid_type = "mean",
  cluster_on_phase = FALSE,
)
```

#### **Arguments**

Either a numeric matrix of data, or an object that can be coerced to such a matrix (such as a numeric vector or a data frame with all numeric columns) or an object of class qts sample.

metric

A character string specifying the distance measure to be used. This must be one of "euclidean", "maximum", "manhattan", "canberra", "binary" or "minkowski" if x is not a QTS sample. Otherwise, it must be one of "12", "pearson" or "dtw".

linkage\_criterion

A string specifying which linkage criterion should be used to compute distances between sets of curves. Choices are "complete" for complete linkage, "average" for average linkage and "single" for single linkage. See stats::hclust() for more details. Defaults to "complete".

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... Further graphical arguments. E.g., cex controls the size of the labels (if plotted)

in the same way as text.

n\_clusters An integer value specifying the number of clusters. Defaults to 1L.

warping\_class A string specifying the warping class Choices are "affine", "dilation", "none",

"shift" or "srsf". Defaults to "affine". The SRSF class is the only class

which is boundary-preserving.

centroid\_type A string specifying the type of centroid to compute. Choices are "mean", "median"

"medoid", "lowess" or "poly". Defaults to "mean". If LOWESS appproximation is chosen, the user can append an integer between 0 and 100 as in "lowess20". This number will be used as the smoother span. This gives the proportion of points in the plot which influence the smooth at each value. Larger values give more smoothness. The default value is 10%. If polynomial approximation is chosen, the user can append an positive integer as in "poly3". This number will be used as the degree of the polynomial model. The default value

is 4L.

cluster\_on\_phase

A boolean specifying whether clustering should be based on phase variation or amplitude variation. Defaults to FALSE which implies amplitude variation.

#### Value

An object of class stats::kmeans or stats::hclust or dbscan\_fast if the input x is NOT of class qts\_sample. Otherwise, an object of class qtsclust which is effectively a list with four components:

- qts\_aligned: An object of class qts\_sample storing the sample of aligned QTS;
- qts\_centers: A list of objects of class qts representing the centers of the clusters;
- best\_clustering: An object of class fdacluster::caps storing the results of the best k-mean alignment result among all initialization that were tried.
- call\_name: A string storing the name of the function that was used to produce the clustering structure;
- call\_args: A list containing the exact arguments that were passed to the function call\_name that produced this output.

#### **Examples**

```
out <- hclust(vespa64$igp[1:10], n_clusters = 2)
plot(out)</pre>
```

hemispherize

QTS Hemispherization

#### **Description**

This function ensures that there are no discontinuities in QTS due to quaternion flips since two unit quaternions q and -q encode the same rotation.

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## Usage

```
hemispherize(x)
## S3 method for class 'qts'
hemispherize(x)
## S3 method for class 'qts_sample'
hemispherize(x)
```

## Arguments

Х

An object of class qts or qts\_sample.

#### Value

An object of the same class as the input argument x with no quaternion flip discontinuities.

## **Examples**

```
hemispherize(vespa64$igp[[1]])
hemispherize(vespa64$igp)
```

inverse\_qts

Inverse Operator for qts Objects

## Description

This function implements the pointwise inverse of a quaternion time series.

## Usage

```
inverse_qts(x)
```

#### **Arguments**

x

An object of class qts.

## Value

An object of class qts storing the inverse of x.

```
inverse_qts(vespa64$igp[[1]])
```

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kmeans

QTS K-Means Alignment Algorithm

## **Description**

This function massages the input quaternion time series to feed them into the k-means alignment algorithm for jointly clustering and aligning the input QTS.

## Usage

```
kmeans(x, n_clusters, ...)
## Default S3 method:
kmeans(
  Х,
  n_{clusters} = 1,
  iter_max = 10,
  nstart = 1,
  algorithm = c("Hartigan-Wong", "Lloyd", "Forgy", "MacQueen"),
  trace = FALSE,
)
## S3 method for class 'qts_sample'
kmeans(
 х,
 n_clusters = 1L,
 seeds = NULL,
 seeding_strategy = c("kmeans++", "exhaustive-kmeans++", "exhaustive", "hclust"),
 warping_class = c("affine", "dilation", "none", "shift", "srsf"),
  centroid_type = "mean",
 metric = c("12", "pearson"),
  cluster_on_phase = FALSE,
  use_fence = FALSE,
)
```

## **Arguments**

iter\_max

Χ	Either a numeric matrix of data, or an object that can be coerced to such a matrix
	(such as a numeric vector or a data frame with all numeric columns) or an object of class qts_sample.
n_clusters	An integer value specifying the number of clusters to be look for.
	not used.

An integer value specifying the maximum number of iterations for terminating the k-mean algorithm. Defaults to 10L.

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nstart if centers is a number, how many random sets should be chosen?

algorithm character: may be abbreviated. Note that "Lloyd" and "Forgy" are alternative

names for one algorithm.

trace logical or integer number, currently only used in the default method ("Hartigan-Wong"):

if positive (or true), tracing information on the progress of the algorithm is pro-

duced. Higher values may produce more tracing information.

seeds An integer value or vector specifying the indices of the initial centroids. If an

integer vector, it is interpreted as the indices of the intial centroids and should therefore be of length n\_clusters. If an integer value, it is interpreted as the index of the first initial centroid and subsequent centroids are chosen according to the k-means++ strategy. It can be NULL in which case the argument seeding\_strategy is used to automatically provide suitable indices. Defaults

to NULL.

seeding\_strategy

A character string specifying the strategy for choosing the initial centroids in case the argument seeds is set to NULL. Choices are "kmeans++", "exhaustive-kmeans++" which performs an exhaustive search over the choice of the first centroid, "exhaustive" which tries on all combinations of initial centroids or "hclust" which first performs hierarchical clustering using Ward's linkage criterion to identify initial centroids. Defaults to "kmeans++", which is the fastest strategy.

warping\_class

A string specifying the warping class Choices are "affine", "dilation", "none", "shift" or "srsf". Defaults to "affine". The SRSF class is the only class which is boundary-preserving.

centroid\_type

A string specifying the type of centroid to compute. Choices are "mean", "median" "medoid", "lowess" or "poly". Defaults to "mean". If LOWESS appproximation is chosen, the user can append an integer between 0 and 100 as in "lowess20". This number will be used as the smoother span. This gives the proportion of points in the plot which influence the smooth at each value. Larger values give more smoothness. The default value is 10%. If polynomial approximation is chosen, the user can append an positive integer as in "poly3". This number will be used as the degree of the polynomial model. The default value is 4L.

metric

A string specifying the metric used to compare curves. Choices are "12" or "pearson". Defaults to "12". Used only when warping\_class != "srsf". For the boundary-preserving warping class, the L2 distance between the SRSFs of the original curves is used.

cluster\_on\_phase

A boolean specifying whether clustering should be based on phase variation or amplitude variation. Defaults to FALSE which implies amplitude variation.

use\_fence

A boolean specifying whether the fence algorithm should be used to robustify the algorithm against outliers. Defaults to FALSE. This is used only when warping\_class != "srsf".

#### Value

An object of class stats::kmeans or stats::hclust or dbscan\_fast if the input x is NOT of class qts\_sample. Otherwise, an object of class qtsclust which is effectively a list with four components:

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- qts\_aligned: An object of class qts\_sample storing the sample of aligned QTS;
- qts\_centers: A list of objects of class qts representing the centers of the clusters;
- best\_clustering: An object of class fdacluster::caps storing the results of the best k-mean alignment result among all initialization that were tried.
- call\_name: A string storing the name of the function that was used to produce the clustering structure;
- call\_args: A list containing the exact arguments that were passed to the function call\_name that produced this output.

#### **Examples**

```
out <- kmeans(vespa64$igp[1:10], n_clusters = 2)</pre>
```

log

QTS Logarithm

## **Description**

This function computes the logarithm of quaternion time series as the time series of the quaternion logarithms.

#### Usage

```
## S3 method for class 'qts'
log(x, ...)
## S3 method for class 'qts_sample'
log(x, ...)
```

#### **Arguments**

x An object of class qts or qts\_sample.

... Extra arguments to be passed on to next methods.

#### Value

An object of the same class as the input argument x in which quaternions have been replaced by their logarithm.

```
log(vespa64$igp[[1]])
log(vespa64$igp)
```

mean.qts\_sample 21

mean.qts\_sample

QTS Geometric Mean

## **Description**

This function computes the pointwise geometric mean of a QTS sample.

## Usage

```
## S3 method for class 'qts_sample'
mean(x, ...)
```

## **Arguments**

x An object of class qts\_sample.

. . . Further arguments passed to or from other methods.

#### Value

An object of class qts in which quaternions are the pointwise geometric mean of the input QTS sample.

#### **Examples**

```
mean(vespa64$igp)
```

median.qts\_sample

QTS Geometric Median

## **Description**

This function computes the pointwise geometric median of a QTS sample.

## Usage

```
## S3 method for class 'qts_sample'
median(x, na.rm = FALSE, ...)
```

## **Arguments**

x An object of class qts\_sample.

na.rm A logical value indicating whether NA values should be stripped before the com-

putation proceeds.

. . . Further arguments passed to or from other methods.

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## Value

An object of class qts in which quaternions are the pointwise geometric median of the input QTS sample.

## **Examples**

```
median(vespa64$igp)
```

moving\_average

QTS Moving Average

## Description

This function performs QTS smoothing via moving average.

## Usage

```
moving_average(x, window_size = 0)
## S3 method for class 'qts'
moving_average(x, window_size = 0)
## S3 method for class 'qts_sample'
moving_average(x, window_size = 0)
```

## **Arguments**

x An object of class qts or qts\_sample.

window\_size An integer value specifying the size of the sliding window used to compute the

median value. Defaults to 0L.

#### Value

An object of the same class as the input argument x storing the smoothed QTS.

```
moving_average(vespa64$igp[[1]], window_size = 5)
moving_average(vespa64$igp, window_size = 5)
```

normalize 23

normalize

QTS Normalization

## **Description**

This function ensures that all quaternions in the time series are unit quaternions.

## Usage

```
normalize(x)
## S3 method for class 'qts'
normalize(x)
## S3 method for class 'qts_sample'
normalize(x)
```

#### **Arguments**

Х

An object of class qts or qts\_sample.

#### Value

An object of the same class as the input argument x in which quaternions are unit quaternions.

## **Examples**

```
normalize(vespa64$igp[[1]])
normalize(vespa64$igp)
```

plot.prcomp\_qts

Plot for prcomp\_qts objects

## **Description**

This function creates a visualization of the results of the PCA applied on a sample of QTS without returning the plot data as an object.

## Usage

```
## S3 method for class 'prcomp_qts'
plot(x, what = "PC1", ...)
## S3 method for class 'prcomp_qts'
screeplot(x, ...)
```

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#### **Arguments**

x An object of class prcomp\_qts as produced by the prcomp.qts\_sample() method.

what

A string specifying what kind of visualization the user wants to perform. Choices are words starting with PC and ending with a PC number (in which case the mean QTS is displayed along with its perturbations due to the required PC) or scores (in which case individuals are projected on the required plane). Defaults to PC1.

. . .

If what = "PC?", the user can specify whether to plot the QTS in the tangent space or in the original space by providing a boolean argument original\_space which defaults to TRUE. If what = "scores", the user can specify the plane onto which the individuals will be projected by providing a length-2 integer vector argument plane which defaults to 1:2.

#### Value

No return value, called for side effects.

#### **Examples**

```
df <- as_qts_sample(vespa64$igp[1:16])
res_pca <- prcomp(df)

# You can plot the effect of a PC on the mean
plot(res_pca, what = "PC1")

# You can plot the data points in a PC plane
plot(res_pca, what = "scores")</pre>
```

plot.qts

Plot for qts objects

#### Description

This function creates a visualization of a QTS without returning the plot data as an object.

## Usage

```
## S3 method for class 'qts'
plot(x, highlighted_points = NULL, ...)
```

#### **Arguments**

x An object of class qts.

highlighted\_points

An integer vector specifying point indices to be highlighted. Defaults to NULL, in which case no point will be highlighted with respect to the others.

. . . Further arguments to be passed on to next methods.

plot.qtsclust 25

## Value

No return value, called for side effects.

## Examples

```
plot(vespa64$igp[[1]])
```

plot.qtsclust

*Plot for* qtsclust *objects* 

## Description

This function creates a visualization of the clustering results obtained on a sample of QTS **without** returning the plot data as an object.

## Usage

```
## S3 method for class 'qtsclust' plot(x, ...)
```

## Arguments

- x An object of class qtsclust as produced by kmeans.qts\_sample() or hclust.qts\_sample().
- . . . Further arguments to be passed to other methods.

## Value

No return value, called for side effects.

```
out <- kmeans(vespa64$igp[1:10], n_clusters = 2)
plot(out)</pre>
```

26 prcomp.qts\_sample

|--|

## Description

This function creates a visualization of a sample of QTS without returning the corresponding gg-plot2::ggplot object

## Usage

```
## S3 method for class 'qts_sample'
plot(x, memberships = NULL, highlighted = NULL, with_animation = FALSE, ...)
```

## Arguments

X	An object of class qts_sample.
memberships	A vector coercible as factor specifying a group membership for each QTS in the sample. Defaults to NULL, in which case no grouping structure is displayed.
highlighted	A boolean vector specifying whether each QTS in the sample should be hightlighted. Defaults to NULL, in which case no QTS is hightlighted w.r.t. the others.
with_animation	A boolean value specifying whether to create a an animated plot or a static gg-plot2::ggplot object. Defaults to FALSE which will create a static plot.
	Further arguments to be passed to methods.

## Value

No return value, called for side effects.

## **Examples**

```
plot(vespa64$igp)
```

## Description

This is the S3 specialization of the function stats::prcomp() for QTS samples.

## Usage

```
## S3 method for class 'qts_sample'
prcomp(x, M = 5, fit = FALSE, ...)
```

predict.prcomp\_qts 27

## Arguments

X	An object of class qts_sample.
М	An integer value specifying the number of principal component to compute. Defaults to $5L$ .
fit	A boolean specifying whether the resulting prcomp_qts object should store a reconstruction of the sample from the retained PCs. Defaults to FALSE.
	Arguments passed to or from other methods.

#### **Details**

The mean\_qts component of the resulting object is the QTS used for centering. It it part of the prcomp\_qts object because it is needed to reconstruct the sample from the retained PCs. The prcomp\_qts object also contains the total variance of the sample and the percentage of variance explained by each PC.

#### Value

An object of class prcomp\_qts which is a list with the following components:

- tpca: An object of class MFPCAfit as produced by the function MFPCA::MFPCA(),
- var\_props: A numeric vector storing the percentage of variance explained by each PC,
- total\_variance: A numeric value storing the total variance of the sample,
- mean\_qts: An object of class qts containing the mean QTS (used for centering the QTS sample before projecting it to the tangent space),
- principal\_qts: A list of qtss containing the required principal components.

## **Examples**

```
res_pca <- prcomp(vespa64$igp[1:16])
```

## **Description**

This function predicts the QTS of a new sample from the PCA decomposition of a previous sample.

## Usage

```
## S3 method for class 'prcomp_qts'
predict(object, newdata, ...)
```

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## **Arguments**

object An object of class prcomp\_qts as produced by the prcomp.qts\_sample() method.

newdata An object of class qts or qts\_sample specifying a QTS or a sample of QTS.

The QTS should be evaluated on the same grid as the one used to fit the PCA model. If the evaluation grids map the same domain but with different sampling frequenciesa, the QTS will be linearly interpolated (in the Lie algebra) to the

common grid used to fit the PCA model.

... Additional arguments. Not used here.

#### Value

An object of class qts\_sample containing the predicted QTS.

## **Examples**

```
# Fit PCA model
pr <- prcomp(vespa64$igp, M = 5)
# Predict QTS
new_qts <- predict(pr)</pre>
```

qts

QTS Class

#### **Description**

A collection of functions that implements the QTS class. It currently provides the as\_qts() function for QTS coercion of tibble::tibbles and the is\_qts() function for checking if an object is a QTS.

## Usage

```
as_qts(x)
is_qts(x)
## S3 method for class 'qts'
format(x, digits = 5, ...)
```

#### **Arguments**

x A tibble::tibble with columns time, w, x, y and z.

digits An integer value specifying the number of digits to keep for printing. Defaults

to 5L.

. . . Further arguments passed to or from other methods.

qts2aats 29

#### **Details**

A quaternion time series (QTS) is stored as a tibble::tibble with 5 columns:

- time: A first column specifying the time points at which quaternions were collected;
- w: A second column specifying the first coordinate of the collected quaternions;
- x: A third column specifying the second coordinate of the collected quaternions;
- y: A fourth column specifying the third coordinate of the collected quaternions;
- z: A fifth column specifying the fourth coordinate of the collected quaternions.

#### Value

An object of class qts.

#### **Examples**

```
qts1 <- vespa64$igp[[1]]
qts2 <- as_qts(qts1)
is_qts(qts1)
is_qts(qts2)</pre>
```

qts2aats

QTS Transformation to Angle-Axis Time Series

## **Description**

This function converts a quaternion time series into its angle-axis representation.

## Usage

```
qts2aats(x)
```

## **Arguments**

Х

An object of class qts.

#### Value

A time series stored as a tibble::tibble with columns time, angle, ux, uy and uz containing the angle-axis representation of the input quaternions.

```
qts2aats(vespa64$igp[[1]])
```

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qts2ats

QTS Transformation To Angle Time Series

## **Description**

This function computes a univariate time series representing the angle between the first and other attitudes.

## Usage

```
qts2ats(x, disable_normalization = FALSE)
```

## **Arguments**

x An object of class qts.

disable\_normalization

A boolean specifying whether quaternion normalization should be disabled. Defaults to FALSE.

#### Value

A time series stored as a tibble::tibble with columns time and angle in which angle measures the angle between the current rotation and the first one.

## **Examples**

```
qts2ats(vespa64$igp[[1]])
```

qts2avts

QTS Transformation to Angular Velocity Time Series

## Description

This function projects a quaternion time series into the space of angular velocities.

## Usage

```
qts2avts(x, body_frame = FALSE)
```

#### **Arguments**

x An object of class qts.

body\_frame A boolean specifying whether the fixed frame with respect to which coordinates

of the angular velocity should be computed is the body frame or the global

frame. Defaults to FALSE.

qts2dts 31

## Value

A time series stored as a tibble::tibble with columns time, x, y and z containing the angular velocity at each time point.

## **Examples**

```
qts2avts(vespa64$igp[[1]])
```

qts2dts

QTS Transformation To Distance Time Series

## Description

This function computes a real-valued time series reporting the pointwise geodesic distance between the two input QTS at each time point.

## Usage

```
qts2dts(x, y)
```

## Arguments

x An object of class qts.

y An object of class qts.

#### **Details**

The function currently expects that the two input QTS are evaluated on the same time grid.

#### Value

A time series stored as a tibble::tibble with columns time and distance in which distance measures the angular distance between the quaternions of both input QTS at a given time point.

```
qts2dts(vespa64$igp[[1]], vespa64$igp[[2]])
```

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qts2nts

QTS Transformation To Norm Time Series

## **Description**

This function computes a univariate time series representing the norm of the quaternions.

#### Usage

```
qts2nts(x, disable_normalization = FALSE)
```

## **Arguments**

```
x An object of class qts. disable_normalization
```

A boolean specifying whether quaternion normalization should be disabled. Defaults to FALSE.

#### Value

A time series stored as a tibble::tibble with columns time and norm in which norm measures the angular distance between the current quaternion and the identity.

#### **Examples**

```
qts2nts(vespa64$igp[[1]])
```

qts\_sample

QTS Sample Class

## Description

A collection of functions that implements the QTS sample class. It currently provides the as\_qts\_sample() function for QTS sample coercion of lists of qts objects, the is\_qts\_sample() function for checking if an object is a QTS sample and the subset operator.

#### Usage

```
as_qts_sample(x)
is_qts_sample(x)
## S3 method for class 'qts_sample'
x[i, simplify = FALSE]
```

reorient 33

#### **Arguments**

X	A list of tibble::tibbles, each of which with columns time, w, x, y and z.
i	A valid expression to subset observations from a QTS sample.
simplify	A boolean value specifying whether the resulting subset should be turned into a single OTS in case the subset is of size 1. Defaults to FALSE.

#### **Details**

A QTS sample is a collection of quaternion time series (QTS), each of which is stored as a tibble::tibble with 5 columns:

- time: A first column specifying the time points at which quaternions were collected;
- w: A second column specifying the first coordinate of the collected quaternions;
- x: A third column specifying the second coordinate of the collected quaternions;
- y: A fourth column specifying the third coordinate of the collected quaternions;
- z: A fifth column specifying the fourth coordinate of the collected quaternions.

#### Value

An object of class qts\_sample.

#### **Examples**

```
x <- vespa64$igp
y <- as_qts_sample(x)
is_qts_sample(x)
is_qts_sample(y)
x[1]
x[1, simplify = TRUE]</pre>
```

reorient

QTS Reorientation

## **Description**

This function reorients the quaternions in a QTS for representing attitude with respect to the orientation of the sensor at the first time point.

## Usage

```
reorient(x, disable_normalization = FALSE)
## S3 method for class 'qts'
reorient(x, disable_normalization = FALSE)
## S3 method for class 'qts_sample'
reorient(x, disable_normalization = FALSE)
```

34 resample

## **Arguments**

```
x An object of class qts or qts_sample. disable_normalization
```

A boolean specifying whether quaternion normalization should be disabled. Defaults to FALSE.

#### Value

An object of the same class as the input argument x in which quaternions measure attitude with respect to the orientation of the sensor at the first time point.

#### **Examples**

```
reorient(vespa64$igp[[1]])
reorient(vespa64$igp)
```

resample

QTS Resampling

## **Description**

This function performs uniform resampling using SLERP.

## Usage

```
resample(x, tmin = NA, tmax = NA, nout = 0L, disable_normalization = FALSE)
## S3 method for class 'qts'
resample(x, tmin = NA, tmax = NA, nout = 0L, disable_normalization = FALSE)
## S3 method for class 'qts_sample'
resample(x, tmin = NA, tmax = NA, nout = 0L, disable_normalization = FALSE)
```

#### **Arguments**

X	x An object of class qts or qts_sample.	
tmin	A numeric value specifying the lower bound of the time interval over which uniform resampling should take place. It must satisfy tmin >= min(qts\$time). Defaults to NA in which case it is set to min(qts\$time).	
tmax	A numeric value specifying the upper bound of the time interval over which uniform resampling should take place. It must satisfy tmax <= max(qts\$time). Defaults to NA in which case it is set to max(qts\$time).	
nout	An integer specifying the size of the uniform grid for time resampling. Defaults to 0L in which case it uses the same grid size as the input QTS.	
disable_normalization		

A boolean specifying whether quaternion normalization should be disabled. Defaults to FALSE in which case the function makes sure that quaternions are normalized prior to performing SLERP interpolation.

rnorm\_qts 35

#### Value

An object of the same class as the input argument x in which quaternions are uniformly sampled in the range [tmin, tmax].

#### **Examples**

```
resample(vespa64$igp[[1]])
resample(vespa64$igp)
```

rnorm\_qts

QTS Random Sampling

## **Description**

This function adds uncorrelated Gaussian noise to the logarithm QTS using an exponential covariance function.

#### Usage

```
rnorm_qts(n, mean_qts, alpha = 0.01, beta = 0.001)
```

#### **Arguments**

n An integer specifying how many QTS should be generated.

mean\_qts An object of class qts specifying the mean QTS.

alpha A positive scalar specifying the variance of each component of the log-QTS.

Defaults to 0.01.

beta A positive scalar specifying the exponential weight. Defaults to 0.001.

#### **Details**

See exp\_cov\_function for details about the roles of alpha and beta in the definition of the covariance operator.

#### Value

A list of n objects of class qts with added noise as specified by parameters alpha and beta.

```
rnorm_qts(1, vespa64$igp[[1]])
```

36 scale

scale

QTS Sample Centering and Standardization

## Description

QTS Sample Centering and Standardization

#### Usage

```
scale(x, center = TRUE, scale = TRUE, ...)
## Default S3 method:
scale(x, center = TRUE, scale = TRUE, ...)
## S3 method for class 'qts_sample'
scale(
    x,
    center = TRUE,
    scale = TRUE,
    by_row = FALSE,
    keep_summary_stats = FALSE,
    ...
)
```

## Arguments

Х		An object coercible into a numeric matrix or an object of class qts_sample representing a sample of observed QTS.
cent	er	A boolean specifying whether to center the sample. If set to FALSE, the original sample is returned, meaning that no standardization is performed regardless of whether argument scale was set to TRUE or not. Defaults to TRUE.
scal	Le	A boolean specifying whether to standardize the sample once it has been centered. Defaults to TRUE.
		Extra arguments passed on to next methods.
by_r	OW	A boolean specifying whether the QTS scaling should happen for each data point (by_row = TRUE) or for each time point (by_row = FALSE). Defaults to FALSE.
keep	keep_summary_stats	

A boolean specifying whether the mean and standard deviation used for standardizing the data should be stored in the output object. Defaults to FALSE in which case only the list of properly rescaled QTS is returned.

#### Value

A list of properly rescaled QTS stored as an object of class qts\_sample when keep\_summary\_stats = FALSE. Otherwise a list with three components:

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- rescaled\_sample: a list of properly rescaled QTS stored as an object of class qts\_sample;
- mean: a list of numeric vectors storing the corresponding quaternion Fréchet means;
- sd: a numeric vector storing the corresponding quaternion Fréchet standard deviations.

#### **Examples**

```
x <- scale(vespa64$igp)
x[[1]]</pre>
```

smooth

QTS Smoothing via SLERP Interpolation

## **Description**

This function performs a smoothing of a QTS by SLERP interpolation.

#### Usage

```
smooth(x, ...)

## Default S3 method:
smooth(
    x,
    kind = c("3RS3R", "3RSS", "3RSR", "3R", "3", "S"),
    twiceit = FALSE,
    endrule = c("Tukey", "copy"),
    do.ends = FALSE,
    ...
)

## S3 method for class 'qts'
smooth(x, alpha = 0.5, ...)

## S3 method for class 'qts_sample'
smooth(x, alpha = 0.5, ...)
```

#### **Arguments**

An object of class qts or qts\_sample. ... Extra arguments passed on to next methods. kind a character string indicating the kind of smoother required; defaults to "3RS3R". twiceit logical, indicating if the result should be 'twiced'. Twicing a smoother S(y) means S(y) + S(y - S(y)), i.e., adding smoothed residuals to the smoothed values. This decreases bias (increasing variance). endrule a character string indicating the rule for smoothing at the boundary. Either "Tukey" (default) or "copy". 38 straighten

do.ends	logical, indicating if the 3-splitting of ties should also happen at the boundaries
	(ends). This is only used for kind = "S".

alpha A numeric value in [0,1] specifying the amount of smoothing. The closer to

one, the smoother the resulting QTS. Defaults to 0.5.

#### Value

An object of the same class as the input argument x which is a smooth version of the input QTS.

## **Examples**

```
smooth(vespa64$igp[[1]])
smooth(vespa64$igp)
```

straighten

QTS Straightening

## **Description**

This function straightens QTS so that the last point equals the first point.

## Usage

```
straighten(x)
## S3 method for class 'qts'
straighten(x)
## S3 method for class 'qts_sample'
straighten(x)
```

## **Arguments**

Х

An object of class qts or qts\_sample.

#### Value

An object of the same class as the input argument x storing the straightened QTS.

```
straighten(vespa64$igp[[1]])
straighten(vespa64$igp)
```

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vespa

The VESPA dataset

## **Description**

A set of QTS representing individual gait patterns (IGPs) of individuals collected under a number of varying factors.

#### Usage

vespa

#### **Format**

A tibble with 320 rows and 7 columns:

- V: a categorical variable with two levels specifying the ID of the Volunteer;
- E: a categorical variable with two levels specifying the ID of the Experimenter;
- S: a categorical variable with four levels specifying the type of Sensor;
- P: a categorical variable with four levels specifying the Position of the sensor;
- A: a categorical variable with two levels specifying the ID of the Acquisition pathway;
- R: a categorical variable with 5 levels specifying the ID of the Repetition;
- igp: A 101x5 tibble storing a QTS which represents the IGP of the individual under a specific set of VESPA conditions.

#### **Details**

The IGP measures the hip rotation during a typical gait cycle. Each rotation is expressed with respect to the mean position of the sensor during the gait cycle. Each IGP is then straightened so that it is periodic with a last point matching the first one.

vespa64

The VESPA64 dataset

#### Description

A set of QTS representing individual gait patterns (IGPs) of individuals collected under a number of varying factors.

#### Usage

vespa64

vespa64

## **Format**

A tibble with 320 rows and 7 columns:

- V: a categorical variable with two levels specifying the ID of the Volunteer;
- E: a categorical variable with two levels specifying the ID of the Experimenter;
- S: a categorical variable with four levels specifying the type of Sensor;
- P: a categorical variable with four levels specifying the Position of the sensor;
- A: a categorical variable with two levels specifying the ID of the Acquisition pathway;
- igp: A 101x5 tibble storing a QTS which represents the IGP of the individual under a specific set of VESPA conditions.

#### **Details**

The IGP measures the hip rotation during a typical gait cycle. Each rotation is expressed with respect to the mean position of the sensor during the gait cycle. Each IGP is then straightened so that it is periodic with a last point matching the first one.

It is essentially a reduced version of the VESPA data set where IGPs have been averaged over the repetition for each set of conditions.

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